Норма линейного функционала

Является ли функционал линейным и ограниченным? Если да, то найдите его норму.

$$\text{Пространство} \quad c_0 = \left\{ (x_n)_{n=1}^{\infty} : \lim_{n\to\infty} x_n = 0,\ \|x\| = \sup_{n\in\mathbb{N}} |x_n| \right\}.$$

1.1. $f(x) = 8x_3 + 3x_{20} - 3x_{1000}$.

1.2. $f(x) = \sum_{m=1}^{\infty} \frac{x_{2m}}{2^m}$.

1.3. $f(x) = \sum_{m=1}^{\infty} \frac{3^m \cdot m!}{m^m} \cdot x_m$.

1.4. $f(x) = x_3 - x_1 + \sum m^2 \cdot \mathrm{e}^{-\sqrt{m}} \cdot x_m$.

1.5. $f(x) = x_5 - 2x_1 + \sum_{m=1}^{\infty} \frac{\ln m}{m} \cdot x_m$.

1.6. $f(x) = \sum_{m=1}^{\infty} \mathrm{e}^m \left(\frac{m-1}{m+1} \right)^{m(m-1)} x_m$.

1.7. $f(x) = x_{10} - 3x_5 + \sum_{m=2}^{\infty} \frac{1}{m^2 \ln m} x_{2^m}$.

1.8. $f(x) = x_5 - 2x_7 - \sum_{m=1}^{\infty} \frac{1}{\sqrt{m(m+1)(m+2)}} x_m$.

1.9. $f(x) = \sum_{m=1}^{\infty} \frac{\mathrm{e}^m}{2^{3m}} \cdot x_{3m}$.

1.10. $f(x) = \sum_{m=1}^{\infty} \frac{3^m \cdot m}{(m+1)!} \cdot x_{2m}$.

1.11. $f(x) = x_3 - x_1 + \sum_{m=1}^{\infty} m^2 \cdot \mathrm{e}^{-\sqrt{m}} \cdot x_{3m}$

1.12. $f(x) = x_5 - 2x_1 + \sum_{m=1}^{50} \frac{\ln m}{m} \cdot x_m$.

1.13. $f(x) = x_1 - \sum_{m=1}^{\infty} \left(\frac{m-1}{m+1} \right)^{m(m-1)} x_m$.

1.14. $f(x) = x_{10} - 3x_5 + \sum_{m=2}^{\infty} \frac{1}{m^2 \ln m} \cdot x_{m^2}$

1.15. $f(x) = x_5 - 2x_7 - \sum_{m=1}^{100} \frac{1}{\sqrt{m(m+1)}} \cdot x_m$

Пространство $\ell_p = \left\{ (x_n)_{n=1}^{\infty} : \sum_{n=1}^{\infty} \left| x_n \right|^p < \infty,\ \left\| x \right\| = \left(\sum_{n=1}^{\infty} \left| x_n \right|^p \right)^{1/p},\ 1 \leqslant p < \infty \right\}$

1.16. $f(x) = \frac{1}{10} x_{10} - \frac{2}{99} x_{99} + \frac{3}{100} x_{100} - \frac{4}{201} x_{201} \quad (p = 2)$.

1.17. $f(x) = -2x_1 + \sum_{m=1}^{\infty} \frac{x_{2m}}{(2m)!} \quad (p = 1)$.

1.18. $f(x) = -3x_3 + \sum_{m=1}^{100} \mathrm{e} \cdot m! \cdot x_{3m} \quad (p = 2)$.

1.19. $f(x) = -2x_5 + x_6 + \sum_{m=1}^{\infty} \frac{x_{8m}}{m} \quad (p = 3)$.

1.20. $f(x) = x_{100} - x_1 + \sum_{m=1}^{\infty} \frac{x_{2m}}{m! \cdot \mathrm{e}^m} \quad \left(p = \frac{7}{4} \right)$.

1.21. $f(x) = -2x_1 + \sum_{m=1}^{\infty} \frac{x_{2m}}{2m} \quad (p = 3)$.

1.22. $f(x) = -3x_3 + \sum_{m=1}^{100} m! \cdot x_{3m} \quad (p = 4)$.

1.23. $f(x) = -\frac{1}{2} x_5 + x_6 + \sum_{m=1}^{\infty} \frac{x_{8m}}{m^2} \quad (p = 3)$.

1.24. $f(x) = -x_1 + x_{100} + \sum_{m=1}^{\infty} \frac{x_{2m}}{m!} \quad \left(p = \frac{5}{4} \right)$.

1.25. $f(x) = -3x_3 + \sum_{m=2}^{\infty} x_{m^2} \quad (p = 3)$.

1.26. $f(x)=\sum\limits_{m=1}^{\infty}\operatorname{sign}(100-m)\cdot x_m \quad \left(p=\dfrac{7}{3}\right).$

1.27. $f(x)=-3x_3+x_9+\sum\limits_{m=2}^{\infty}x_{m^2} \quad \left(p=1\right).$

1.28. $f(x)=-3x_1+\sum\limits_{m=1}^{10}\sin^2 m\cdot x_m \quad \left(p=2\right).$

Пространство $L_p\left[a,b\right]=\left\{x(t):\left\|x\right\|=\left(\int\limits_a^b\left|x(t)\right|^p\right)^{1/p},\ 1\leqslant p<\infty\right\}.$

1.29. $f(x)=\int\limits_0^{1/2}\sqrt{t}x(t^2)\,dt \quad \left(p=\dfrac{7}{4},\ \left[0,1\right]\right).$

1.30. $f(x)=\int\limits_{-1}^{-1/2}tx(t^2)\,dt-5\int\limits_0^1 x\left(\sqrt{t}\right)dt \quad \left(p=\dfrac{3}{2},\left[-1,1\right]\right).$

1.31. $f(x)=\int\limits_0^1 t^4x(t^3)\,dt \quad \left(p=\dfrac{9}{2},\left[-1,1\right]\right).$

1.32. $f(x)=\int\limits_5^6\sin(\pi t)x\left(\sqrt{t}\right)dt \quad \left(p=1,\ \left[3,9\right]\right).$

1.33. $f(x)=\int\limits_0^1\sqrt{t}x(t^2)\,dt \quad \left(p=9,\left[0,2\right]\right).$

1.34. $f(x)=\int\limits_0^{1/2}\sqrt[3]{t}x\left(\sqrt[3]{t}\right)dt \quad \left(p=\dfrac{6}{5},\left[-1,1\right]\right).$

1.35. $f(x)=\int\limits_0^1\dfrac{1}{\sqrt[3]{t}}x(t^2)\,dt \quad \left(p=1,\left[0,1\right]\right).$

1.36. $f(x)=\int\limits_0^{1/2}\sqrt[3]{t^2}x(t^3)\,dt \quad \left(p=2,\left[-1,1\right]\right).$

1.37. $f(x)=\int\limits_0^{1/2}\sqrt{t}x(t^3)\,dt \quad \left(p=\dfrac{5}{4},\left[0,1\right]\right).$

1.38. $f(x) = \int\limits_{-1}^{-1/2} tx(t^3)\,dt - 2\int\limits_0^1 x\left(\sqrt{t}\right)dt \quad \left(p = 3, \left[-1,1\right]\right).$

1.39. $f(x) = \int\limits_5^6 \sin(\pi t)x(t)\,dt \quad \left(p = 1, \left[3,9\right]\right).$

1.40. $f(x) = \int\limits_0^1 \sqrt{t}x(t^2)\,dt \quad \left(p = 7, \left[0,2\right]\right).$

1.41. $f(x) = \int\limits_0^{1/2} \sqrt[3]{t}x\left(\sqrt[11]{t}\right)dt \quad \left(p = \dfrac{9}{5}, \left[-1,1\right]\right).$

1.42. $f(x) = \int\limits_0^1 \dfrac{1}{\sqrt[3]{t}}x(t^2)\,dt \quad \left(p = 1, \left[0,1\right]\right).$

Пространство $C\left[a,b\right] = \left\{x(t) : \left\|x\right\| = \max\limits_{a<t<b}\left|x(t)\right|\right\}.$

1.43.